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B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2015
FIRST YEAR
MATHEMATICS (Honours)
Paper : II

Date : 21/05/2015
Time : 11 am – 3 pm
Full Marks : 100

[Use a separate Answer book for each Group]

Group – A

Unit - I

[*Answer **any five** questions*] [5×5]

1. State and prove the Cauchy-Schwarz inequality. [1+4]

2. a) If n is a positive integer > 1, prove that $2^{n(n+1)} > (n+1)^{n+1}\left(\frac{n}{1}\right)^n\left(\frac{n-1}{2}\right)^{n-1}\cdots\left(\frac{2}{n-1}\right)^2\frac{1}{n}$. [2]

   b) Prove that the greatest value of a^2b^3 is $\frac{3}{16}$ where a, b are positive real numbers satisfying the condition $3a+4b=5$. [3]

3. If x is real, prove that

$$i\log\frac{x-i}{x+i} = \pi - 2\tan^{-1}x, \text{ if } x>0$$
$$= -\pi - 2\tan^{-1}x, \text{ if } x\le 0.$$ [5]

4. a) Show that the product of all the values of $(1+\sqrt{3}i)^{3/4}$ is 8. [3]

   b) Find the principal value of $(1+i)^{1-i}$. [2]

5. a) Find the equation whose roots are the roots of the equation $x^4-8x^2+8x+6=0$, each diminished by 2. [3]

   b) Find the number of special roots of the equation $x^{90} - 1 = 0$. [2]

6. Solve the equation $2x^4-3x^3-3x-2=0$, given that two roots α,β are connected by the relation $2\alpha+\beta=1$. [5]

7. Solve : $x^3+9x^2+15x-25=0$, by Cardan's Method. [5]

8. Solve the equation $x^4+3x^3+5x^2+4x+2=0$ by Ferrari's method. [5]

Unit - II

[*Answer **any five** questions*] [5×5]

9. a) Test the convergence of the series $\sum_{n=3}^{\infty}\frac{1}{n\log n(\log\log n)}, n\ge 3$. [3]

   b) If the series $\sum_{n=1}^{\infty}a_n^2$ and $\sum_{n=1}^{\infty}b_n^2$ are both convergent, prove that the series $\sum_{n=1}^{\infty}a_nb_n$ is convergent. [2]

10. Show that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ converges to log 2, but the rearranged series $1+\frac{1}{3}-\frac{1}{2}+\frac{1}{5}+\frac{1}{7}-\frac{1}{4}+\ldots$ converges to $\frac{3}{2}\log 2$. [5]

11. a) Let K be a compact subset of $\mathbb{R}$ and $F\subset K$ be a closed set in $\mathbb{R}$. Prove that F is compact in $\mathbb{R}$. [3]

b) Let $\{x_n\}_{n\in\mathbb{N}}$ be a convergent sequence in $\mathbb{R}$ with limit ℓ and S be the range set of the sequence. Prove that $S\cup\{\ell\}$ is a compact subset of $\mathbb{R}$. [2]

12. a) Let $I(\subset\mathbb{R})$ be a closed bounded interval. Prove that every continuous function $f: I\to\mathbb{R}$ is uniformly continuous. [3]

b) Prove the following and disprove the converse : Every Lipschitz function is uniformly continuous. [2]

13. Define a function $f:(1,2)\cup\{3\}\cup(4,5)\to\mathbb{R}$ by $f(x)=[x]$, the greatest integer function. Discuss continuity of f on every point of its domain. If f is continuous on the domain, then check further for uniform continuity. [5]

14. a) Let f be a continuous real function on $\mathbb{R}$. Let $Z(f)=\{p\in\mathbb{R}: f(p)=0\}$. Prove that Z(f) is closed. [2]

b) If $c_0+\frac{c_1}{2}+\frac{c_2}{3}+\ldots+\frac{c_n}{n+1}=0$, where $c_0, c_1, \ldots, c_n$ are real numbers, show that the equation $c_0+c_1x+c_2x^2+\ldots+c_nx^n=0$ has at least one real root between 0 and 1. [3]

15. a) If a function f is such that its derivative f′ is continuous on [a, b] and derivable on (a, b), then show that there exists a number c between a and b such that

$$f(b)=f(a)+(b-a)f'(a)=\frac{1}{2}(b-a)^2f''(c).$$ [3]

b) Find out the series of cos x with the help of Maclaurin's Theorem. [2]

16. a) Examine the function $(\sin x+\cos x)$ for extreme values. [2]

b) If $a_1, a_2, \ldots, a_n$ are all positive real numbers, prove that

$$\lim_{x\to\infty}\{x-\sqrt[n]{(x-a_1)(x-a_2)\ldots(x-a_n)}\}=\frac{a_1+a_2+\ldots+a_n}{n}.$$ [3]

Group – B

Unit - I

[*Answer* ***any four*** *questions*] [4×5]

17. Expand by Laplace method to prove that $\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix}=(a^2+b^2+c^2+d^2)^2$. [5]

18. Prove that the matrix $\frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{pmatrix}$ is orthogonal. Utilise this to solve the equations : $x-2y+2z=2,\ 2x-y-2z=1,\ 2x+2y+z=7$. [2+3]

19. Obtain the fully reduced normal form of the matrix $A=\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}$. Find the non-singular matrices P, Q such that PAQ is the fully reduced normal form. [3+2]

20. Extend the set $\{(1,1,1,1),((1,-1,1,-1)\}$ to basis of $\mathbb{R}^4$. [5]

21. Find $\dim(S\cap T)$, where S and T are subspaces of the vector space $\mathbb{R}^4$ given by $S=\{(x,y,z,w)\in\mathbb{R}^4: x+y+z+w=0\}$, $T=\{(x,y,z,w)\in\mathbb{R}^4: 2x+y-z+w=0\}$. [5]

22. a) Find the coordinates of the vector $\alpha = (0,3,1)$ in $\mathbb{R}^3$ relative to the basis $\{\alpha_1, \alpha_2, \alpha_3\}$ where $\alpha_1 = (1,0,1)$, $\alpha_2 = (0,1,1)$, $\alpha_3 = (1,1,0)$. [3]

b) Find a basis of the vector space $\mathbb{R}^3$ that contains the vectors (1, 2, 1) and (3, 6, 2). [2]

Unit - II

[*Answer* ***any three*** *questions*] [3×10]

23. a) A businessman has the option of investing his money in two plans. Plan A guarantees that each rupee invested will earn seventy paise a year hence while the plan B guarantees that each rupee invested will earn two rupees two year hence. In plan B, only investments for periods that are multiple of two years are allowed. The problem is how he invest ten thousand rupees in order to maximize the earnings at the end of the three years. Formulate the problem as a linear programming model. [5]

b) Reduce the feasible solution $x_1 = 4$, $x_2 = 1$, $x_3 = 3$ of the system of equations $2x_1 - 3x_2 + x_3 = 8$, $x_1 + 2x_2 + 3x_3 = 15$ to one or more basic feasible solutions. [5]

24. a) Prove that the number of basic variables in a transportation problem is at most $(m+n-1)$. [4]

b) Use Big-M method to maximize $z = 6x_1 + 4x_2$ subject to the constraints

$$2x_1 + 3x_2 \le 30$$
$$3x_1 + 2x_2 \le 24$$
$$x_1 + x_2 \ge 3$$
$$x_1, x_2 \ge 0.$$

Show that the solution is not unique. Find two solutions. [6]

25. a) Solve the following Travelling Salesman problem : [5]

	A	B	C	D
A	∞	5	8	4
B	5	∞	7	4
C	8	7	∞	8
D	4	4	8	∞

b) Find the optional solution of the following transportation problem : [5]

	D_1	D_2	D_3	D_4	a_i
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	90
b_j	85	35	50	45	

26. a) A company has 5 jobs to be done. The following matrix shows the return in Rs. of assigning five machines (1,2,3,4,5) to the five jobs (A,B,C,D,E). Assign the five jobs to five machines so as to maximize the expected profit. [5]

	1	2	3	4	5
A	5	11	10	12	4
B	2	4	6	3	5
C	3	1	5	14	6
D	6	14	4	11	7
E	7	9	8	12	5

b) Solve the following problem by two-phase method : [5]

Max Z = $2x_1 + x_2 - x_3$

Subject to, $4x_1 + 6x_2 + 3x_3 \leq 8$

$$3x_1 - 6x_2 - 4x_3 \leq 1$$

$$2x_1 + 3x_2 - 5x_3 \geq 4$$

$$x_1, x_2, x_3 \geq 0$$

27. a) Prove that every extreme point of the convex set of all feasible solutions of the system $Ax = b,\ x \geq 0$ corresponds to a basic feasible solution. [4]

b) Find the basic feasible solutions of the following set of equations : [4]

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8$$

$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$x_1, x_2, x_3, x_4 \geq 0$$

c) Prove that in E^2, the set $S = \{(x_1, x_2) / x_1 - 2x_2 = 2\}$ is a convex set. [2]

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